Data Streams: Where to Go?

PODS 11, Tutorial

S. *Muthu* Muthukrishnan

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- Solution:
  B maintains the running sum s of numbers seen.
  Missing number is $\frac{n(n+1)}{2} - s$.

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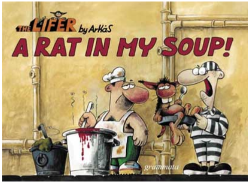

- Solution. Maintain μ_j.
  If $i_{j+1} > \mu_j$, $\mu_{j+1} \leftarrow \mu_j + 1$.
  If $i_{j+1} < \mu_j$, $\mu_{j+1} \leftarrow \mu_j - 1$.

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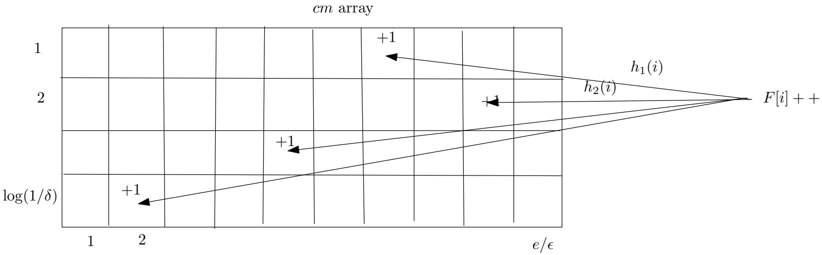
- Query: $F[i] = ?$.

Count-Min Sketch

- For each update $F[i]++$,
  - for each $j = 1, \ldots, \log(1/\delta)$, update $cm[h_j(i)]++$.



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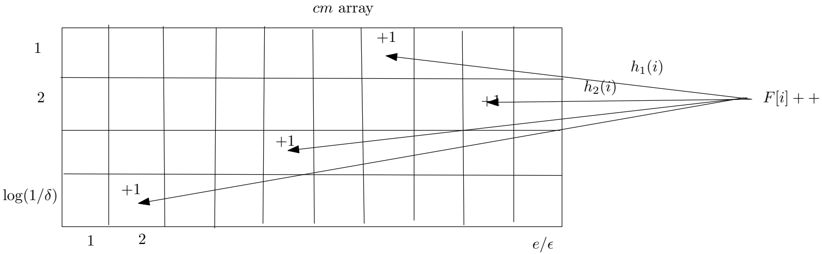

- Estimate $\tilde{F}(i) = \min_{j=1,\ldots,\log(1/\delta)} cm[h_j(i)]$.

Count-Min Sketch

- Claim: $F[i] \leq \tilde{F}[i]$.
- Claim: With probability at least $1 - \delta$, $\tilde{F}[i] \leq F[i] + \varepsilon \sum_{j \neq i} F[j]$.
- Space used is $O(\frac{1}{\varepsilon} \log(\frac{1}{\delta}))$.
- Time per update is $O(\log(\frac{1}{\delta}))$. Indep of n.

G. Cormode and S. Muthukrishnan: An improved data stream summary: count-min sketch and its applications. *Journal of Algorithms*.

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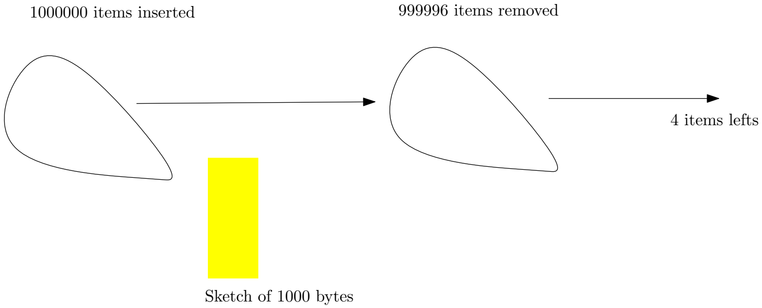
- Consider $\Pr(\tilde{F}[i] > F[i] + \varepsilon \sum_{j \neq i} F[j])$:

$$\begin{aligned} \Pr() &= \Pr(\forall j, F[i] + X_{i,j} > F[i] + \varepsilon \sum_{j \neq i} F[j]) \\ &= \Pr(\forall j, X_{i,j} \geq eE(X_{i,j})) \\ &< e^{-\log(1/\delta)} = \delta \end{aligned}$$

Improve Count-Min Sketch?

- Index Problem:
  - A has n long bitstring and sends messages to B who wishes to compute the ith bit.
  - Needs $\Omega(n)$ bits of communication.
- Reduction of estimating F[i] in data stream model.
  - $I[1 \cdots 1/(2\varepsilon)]$
  - $I[i] = 1 \to F[i] = 2$
  - $I[i] = 0 \to F[i] = 0; F[0] \leftarrow F[0] + 2.$
  - Estimating $F[i]$ to $\varepsilon||F|| = 1$ accuracy reveals $I[i]$.

Count-Min Sketch, The Challenge



- Recovering $F[i]$ to $\pm 0.1|F|$ accuracy will retrieve each item precisely.

Applications of Count-Min Sketch

- Solves many problems in the best possible/known bounds:
  - Data Mining: heavy hitters, heavy hitter differences.
  - Signal processing: Compressed sensing.
  - Statistics: Histograms, Wavelets, Clustering, Least squares.

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Wiki: http://sites.google.com/site/countminsketch/

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- Broken the premise that data has to be
  - captured,
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Polynomial time/space theory -> sublinear theory
Nyquist sampling -> SubNyquist sampling

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Some My-story

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  - Who needs to analyze IP packet data?
- Observation: $1/\varepsilon^2$ space to give ε accuracy. Prohibitive.

Some Successful Streaming Systems

Specialized streaming systems:

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  - Two level archietecture.
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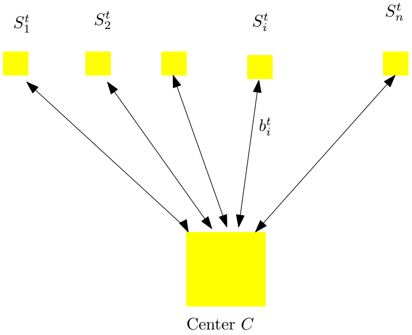
Q: General purpose streaming systems?

Some Research Directions

1: Distributed, continual monitoring



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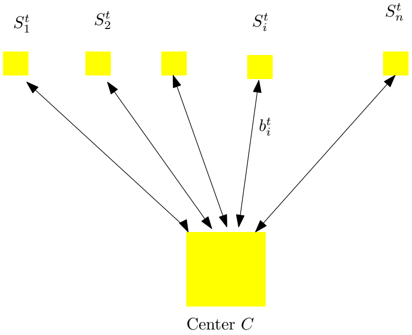

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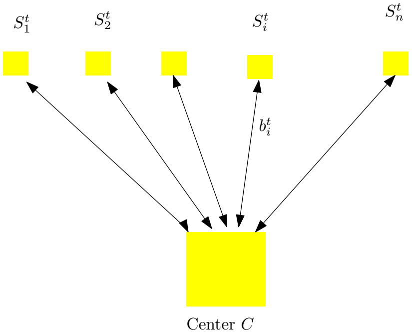

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- Say b_i^t is total number of bits sent b/w i and C
- Minimize $\sum_i b_i^t$.

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- Center outputs 1 when $O(1/\varepsilon^2)$ bits received.
- $O(\frac{1}{\varepsilon^2} \log(\frac{1}{\delta}))$ bits suffice with prob of success $1 - \delta$.
- Independent of k.

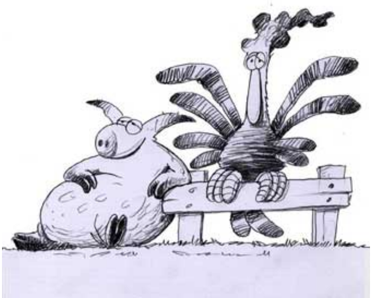
Algorithms for distributed functional monitoring. Cormode, Muthukrishnan, Yi. SODA 08.

1. Distributed, Continual Monitoring: Summary

- Statistics: Frequency moments, Distinct counts.
- Optimization: Clustering.
- Signal processing: Compressed sensing.

Need a fuller theory.
Connections to Slepian-Wolf, network coding.

2. Probabilistic Streams



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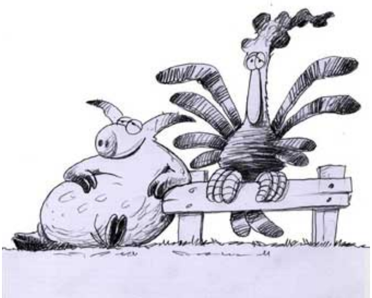

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- The query is to estimate $\Pr[\sum_i X_i \leq c]$.

Probabilistic Streams Contd

Berry-Esseen Theorem

Let $X_1, \ldots, X_n$ be i.i.d. random variables with

- $E(X_i) = 0, E(X^2) = \sigma^2$, and $E(|X|^3) = \rho$.

Let $Y_n = \sum_i X_i / n$ with

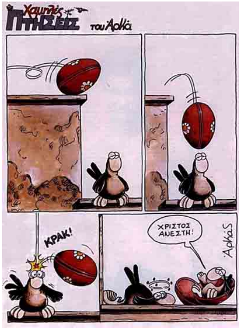
- F_n the cdf of $Y_n \sqrt{n} / \sigma$

ϕ the cdf of the std normal dist.

Then there exists a positive C such that for all x and n,

$$|F_n(x) - \phi(x)| \leq \frac{C\rho}{\sigma^3 \sqrt{n}}.$$

Probabilistic Streams Contd



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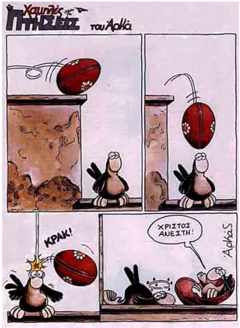

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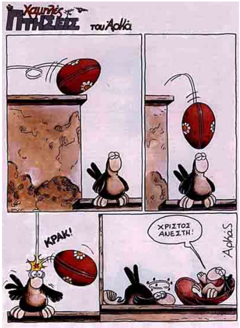

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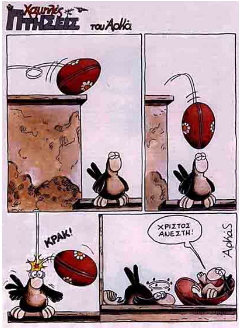

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- This can be approximated by $\phi(c/\sigma\sqrt{n})$.
- To finish up. Estimate σ and its impact on overall error. Extend to more general X_i's.

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- Observe:
  - Can *a priori* look at the dist of $\max_i X_i$.
  - Not the same as finding $\max_i X_i$.

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- Algorithm:
  - $X^* = \max_i X_i$.
  - m: median of X^*, ie., $\Pr(X^* < m) \approx 1/2$.
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- Algorithm finds t such that $E(X_t)/E(OPT) \geq 1/2$.

  Prophet inequality.

Many basic problems on stochastic streams still open.

Conclusions

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  - count-min sketch and applications.
  - classical streaming.
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  - Left out: window streams, rich queries and data, MapReduce